Quantum mechanics on the human scale

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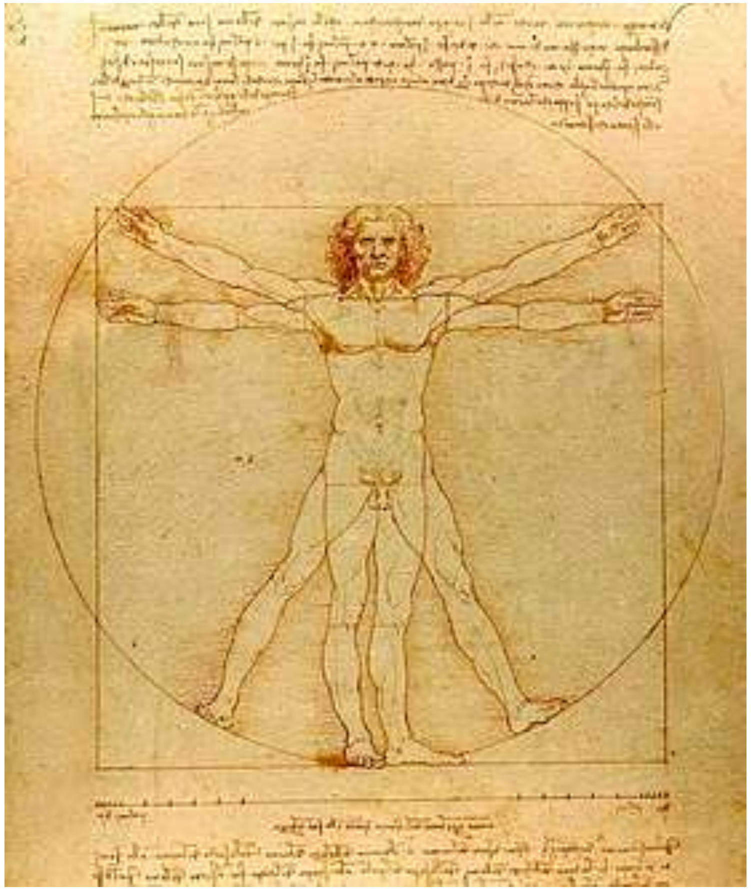
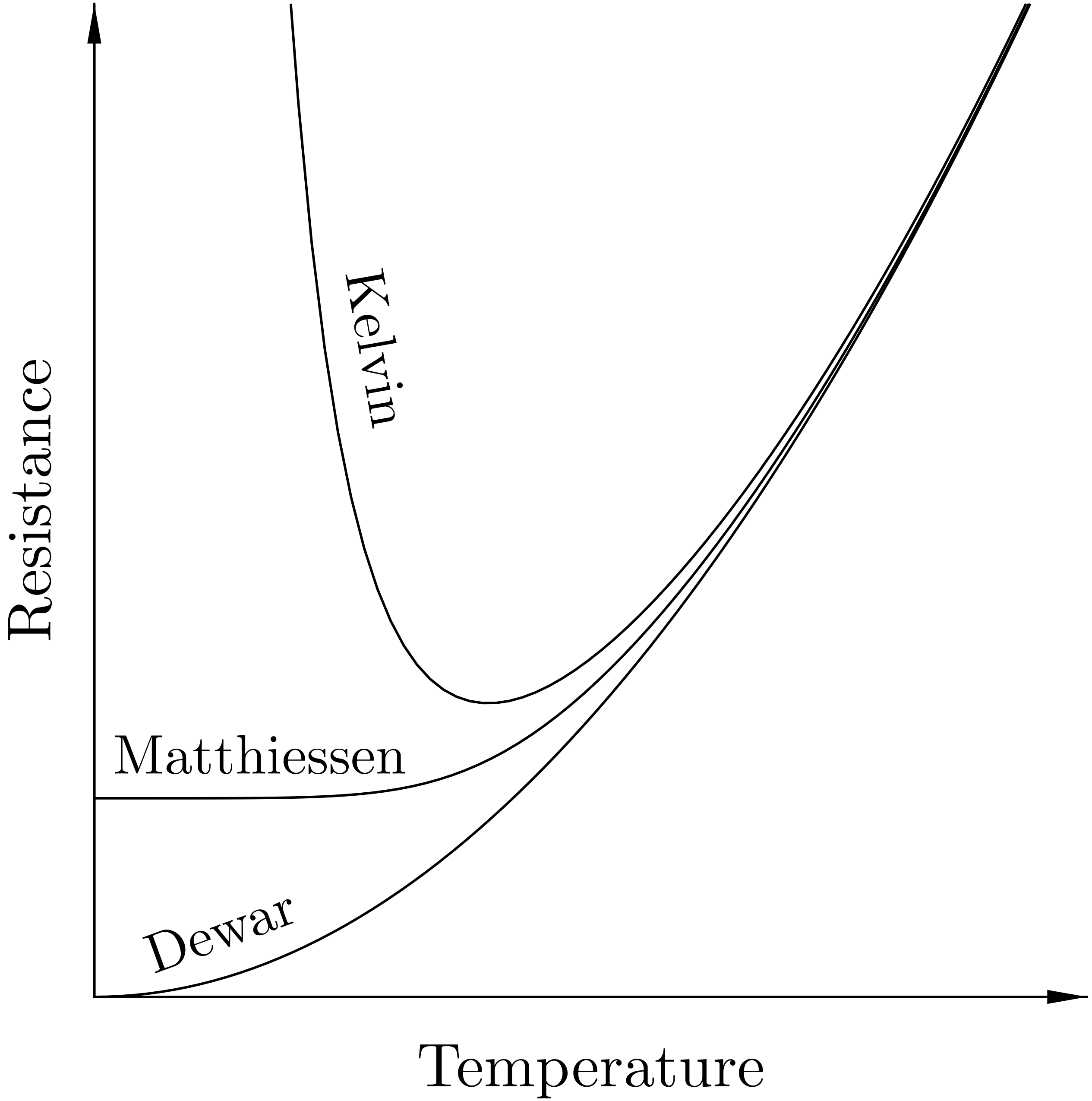
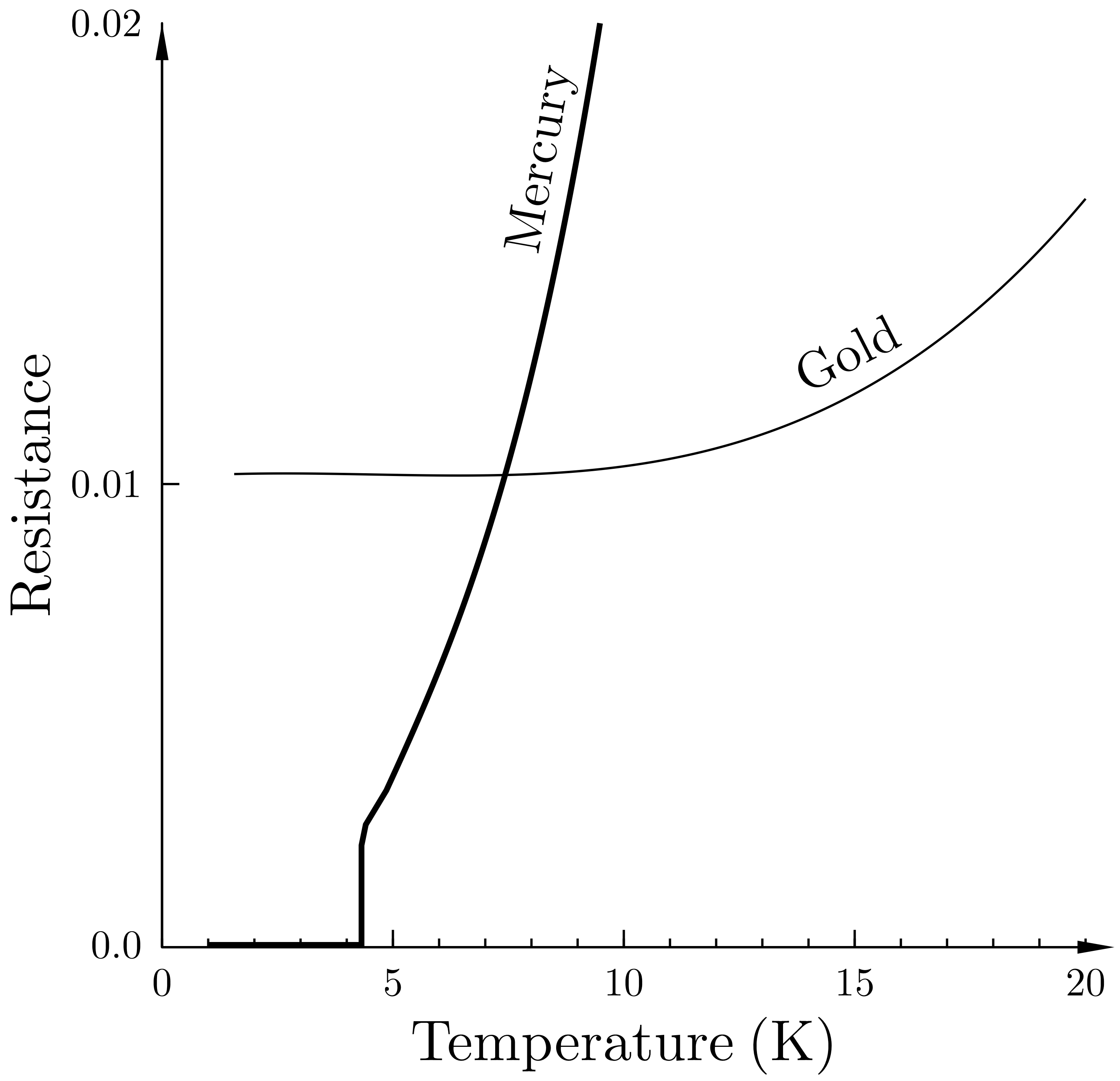


Morning of theoretical physics
Saturday October 28th 2017


SUPERCONDUCTIVITY

Kammerlingh Onnes (1853-1926)

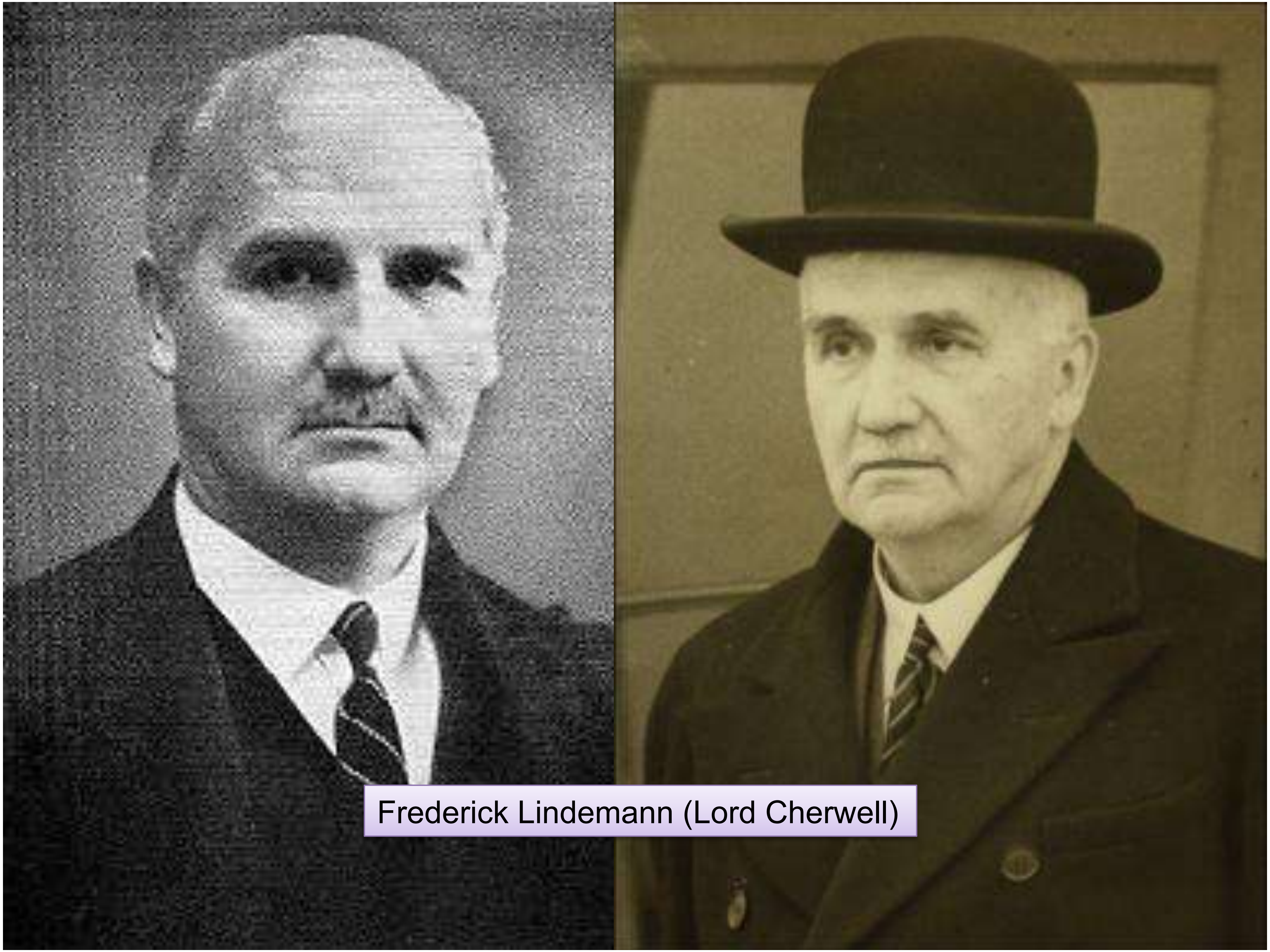

Resistance
Kelvin
Matthiessen
Dewar
Temperature



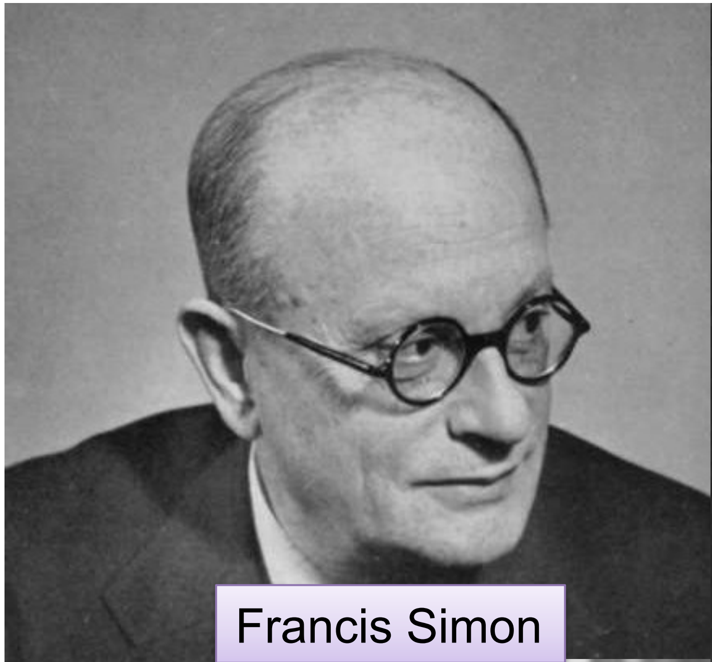
0.02
0.01
0.0
Resistance
Mercury
Gold
0
5
10
15
20
Temperature (K)

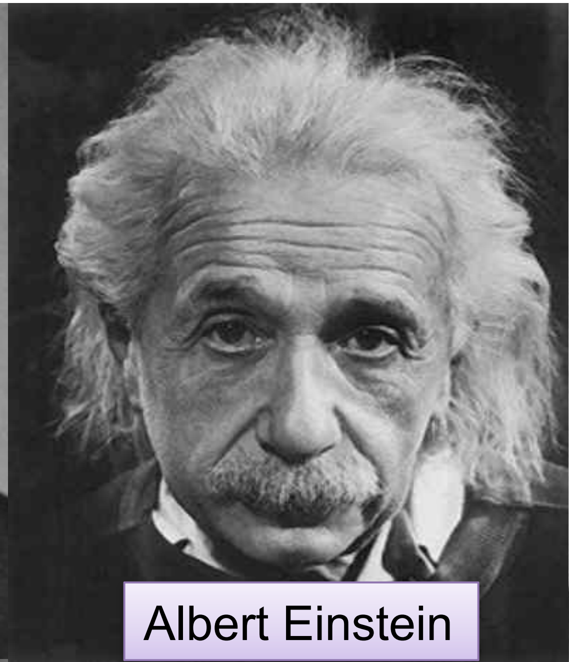
Meissner effect

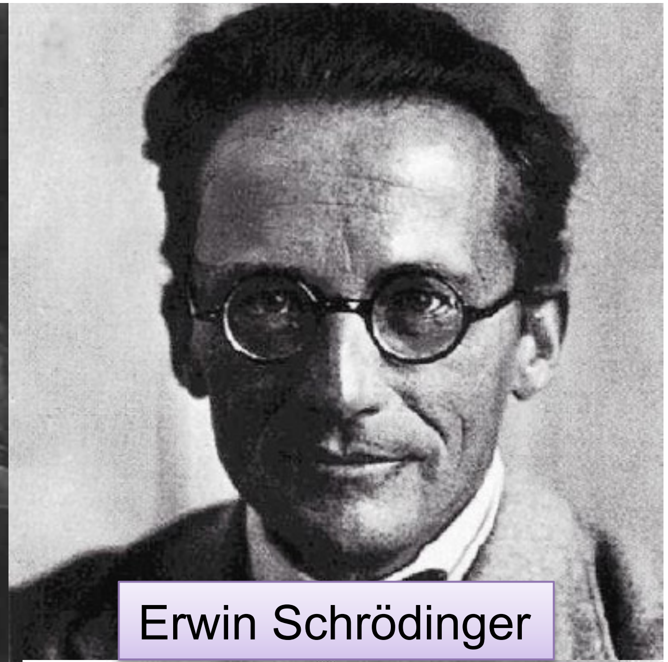
Fritz London (1900-1954) &
Heinz London (1907-1970)

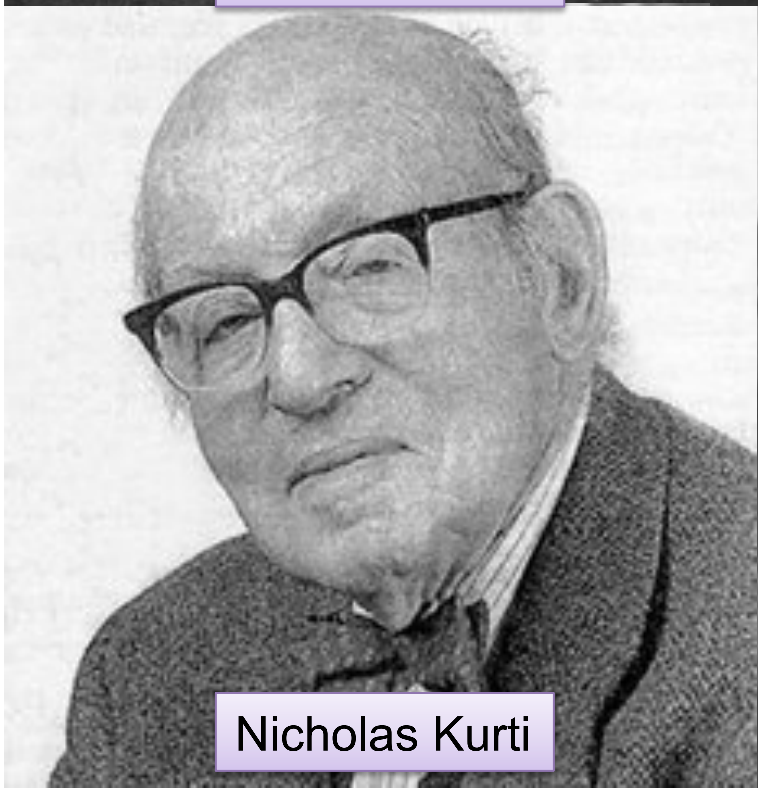


Frederick Lindemann (Lord Cherwell)

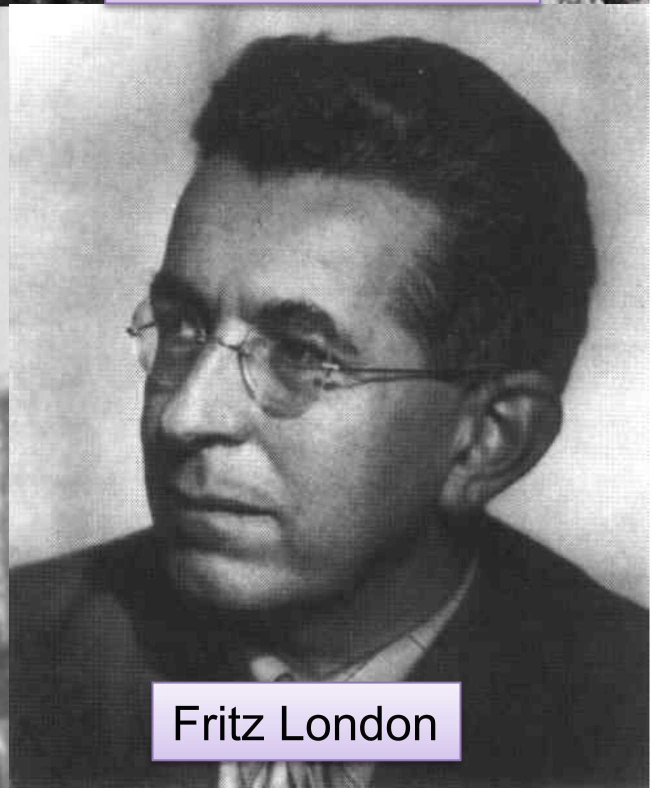

Francis Simon


Albert Einstein

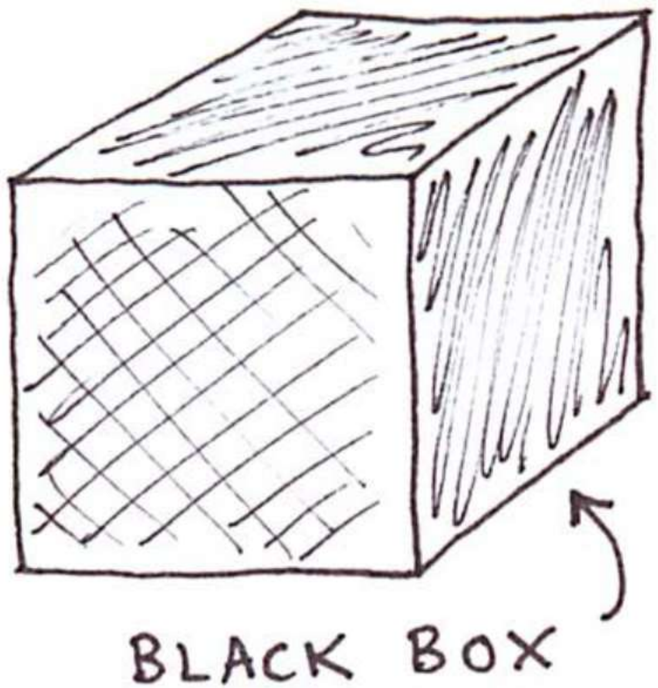

Erwin Schrödinger


Nicholas Kurti

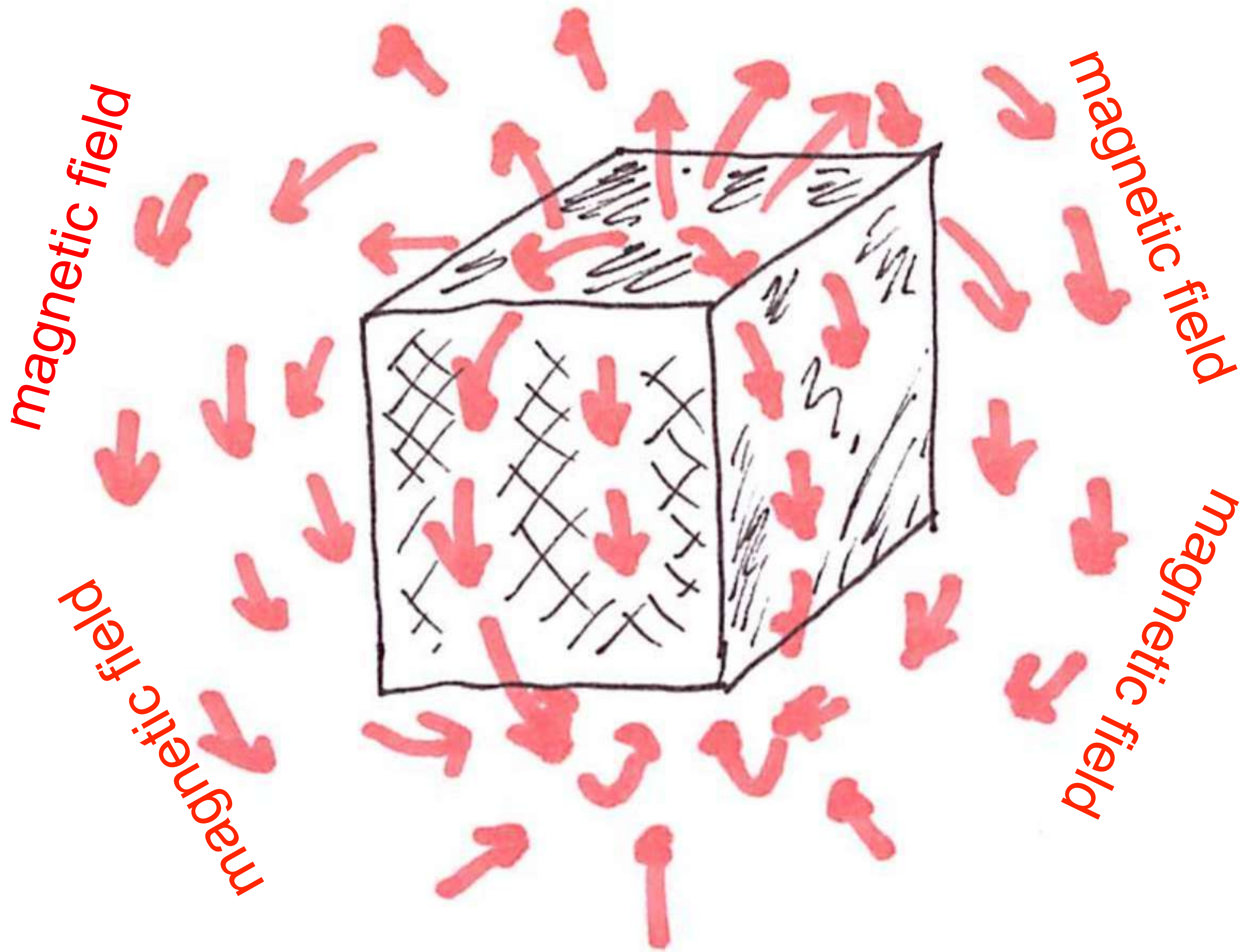

Kurt Mendelssohn


Fritz London

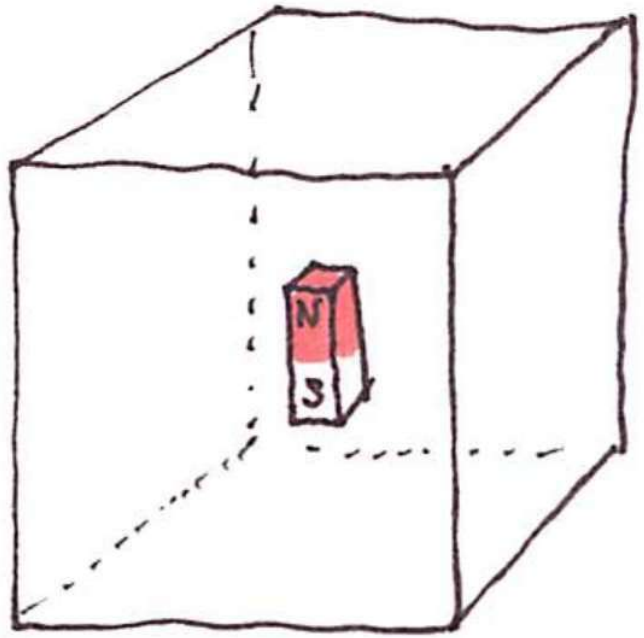
Thought experiment



Thought experiment

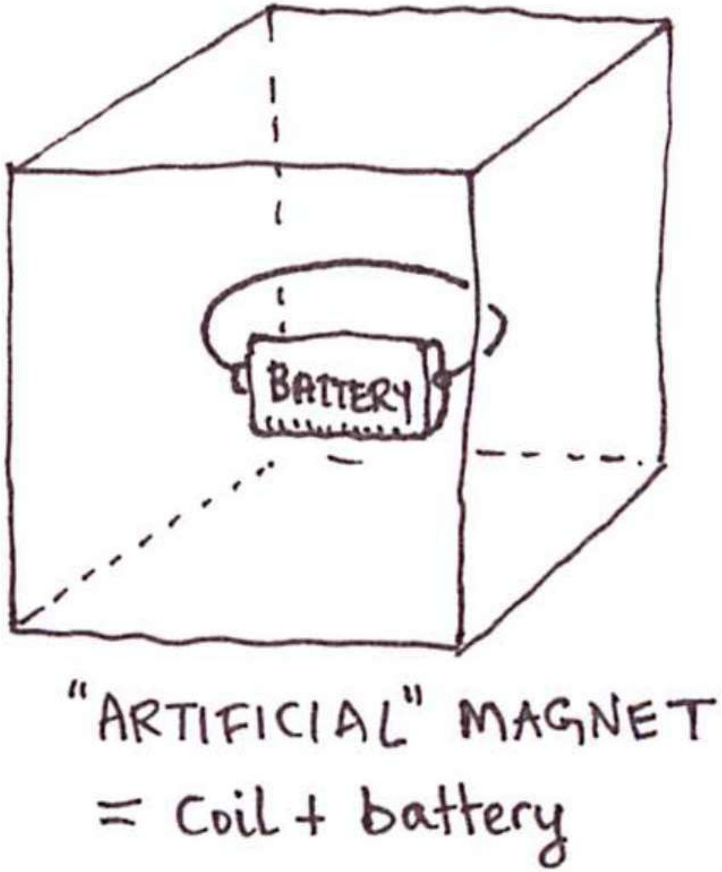


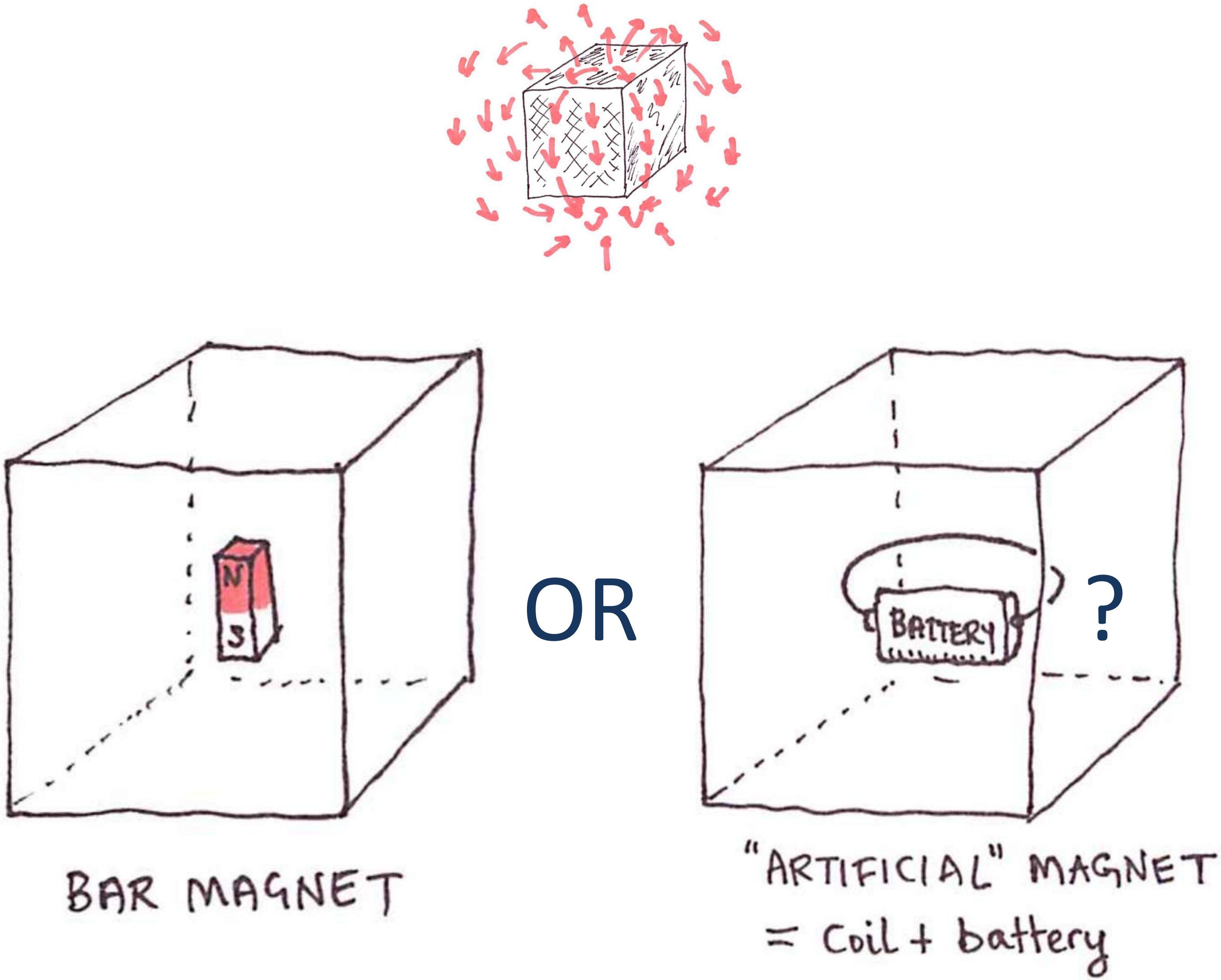
Thought experiment



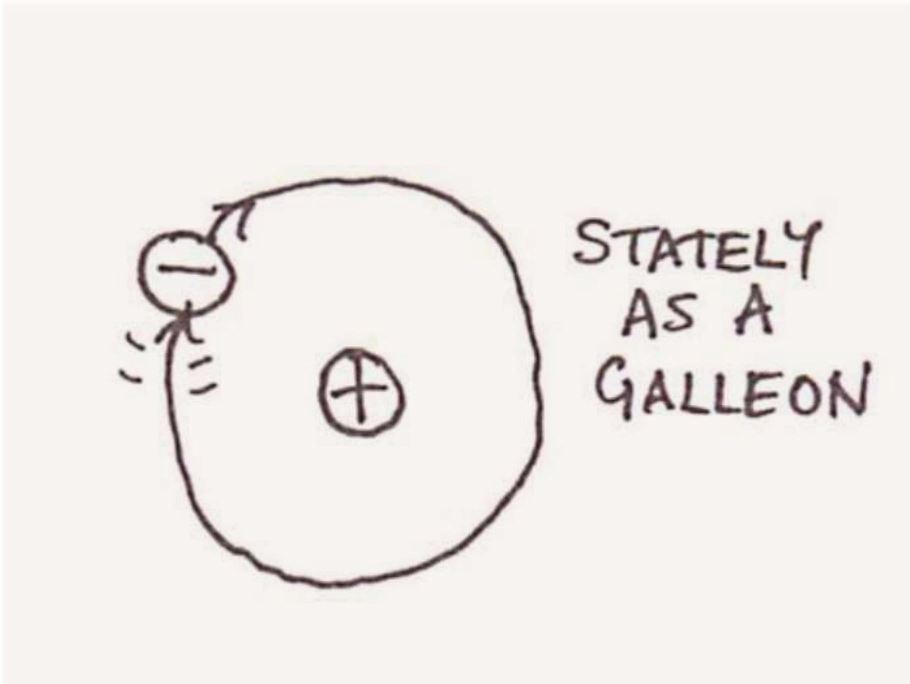
BAR MAGNET

Thought experiment

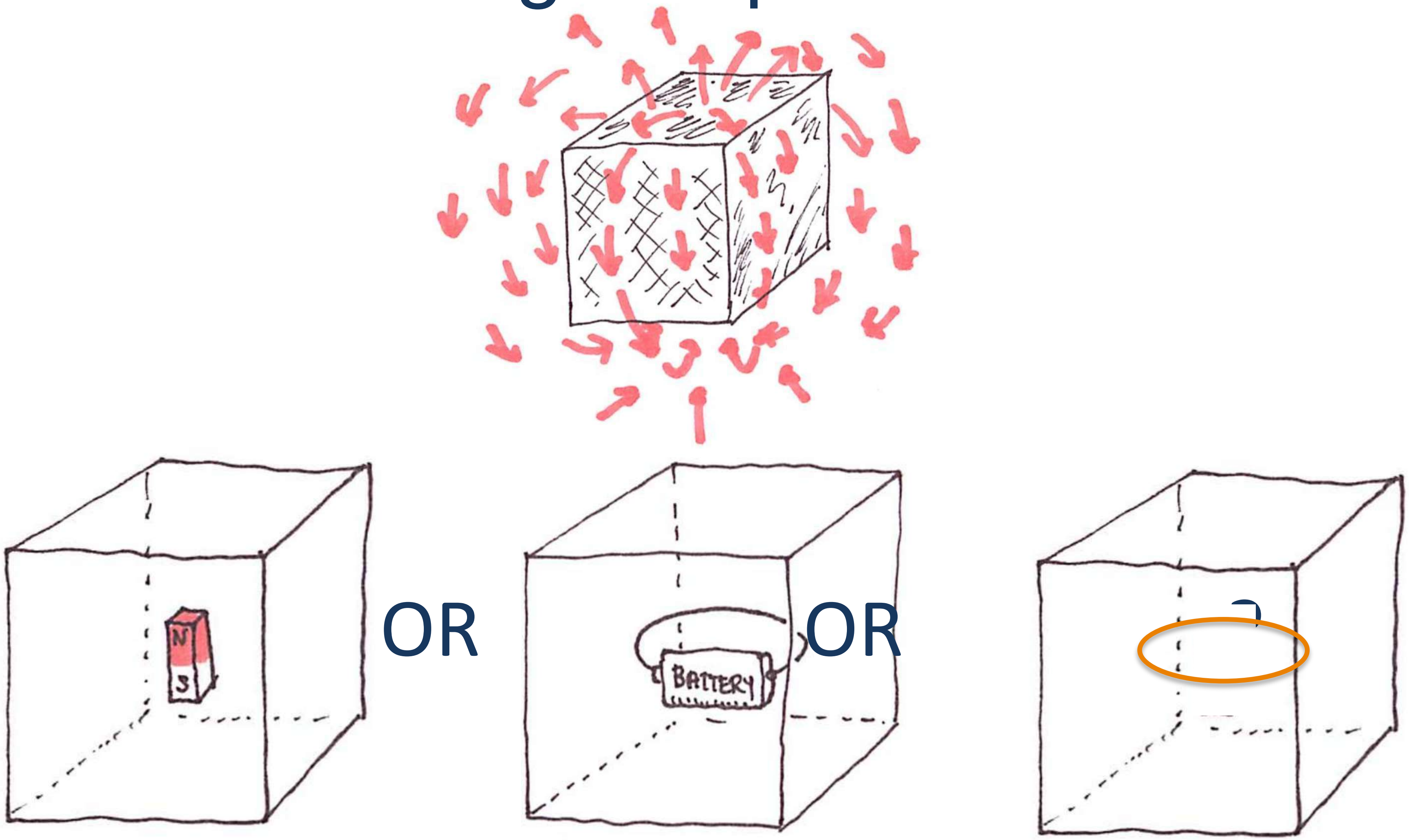



N
S
OR
BATTERY
?
BAR MAGNET
"ARTIFICIAL" MAGNET
= Coil + battery

The atom



Thought experiment



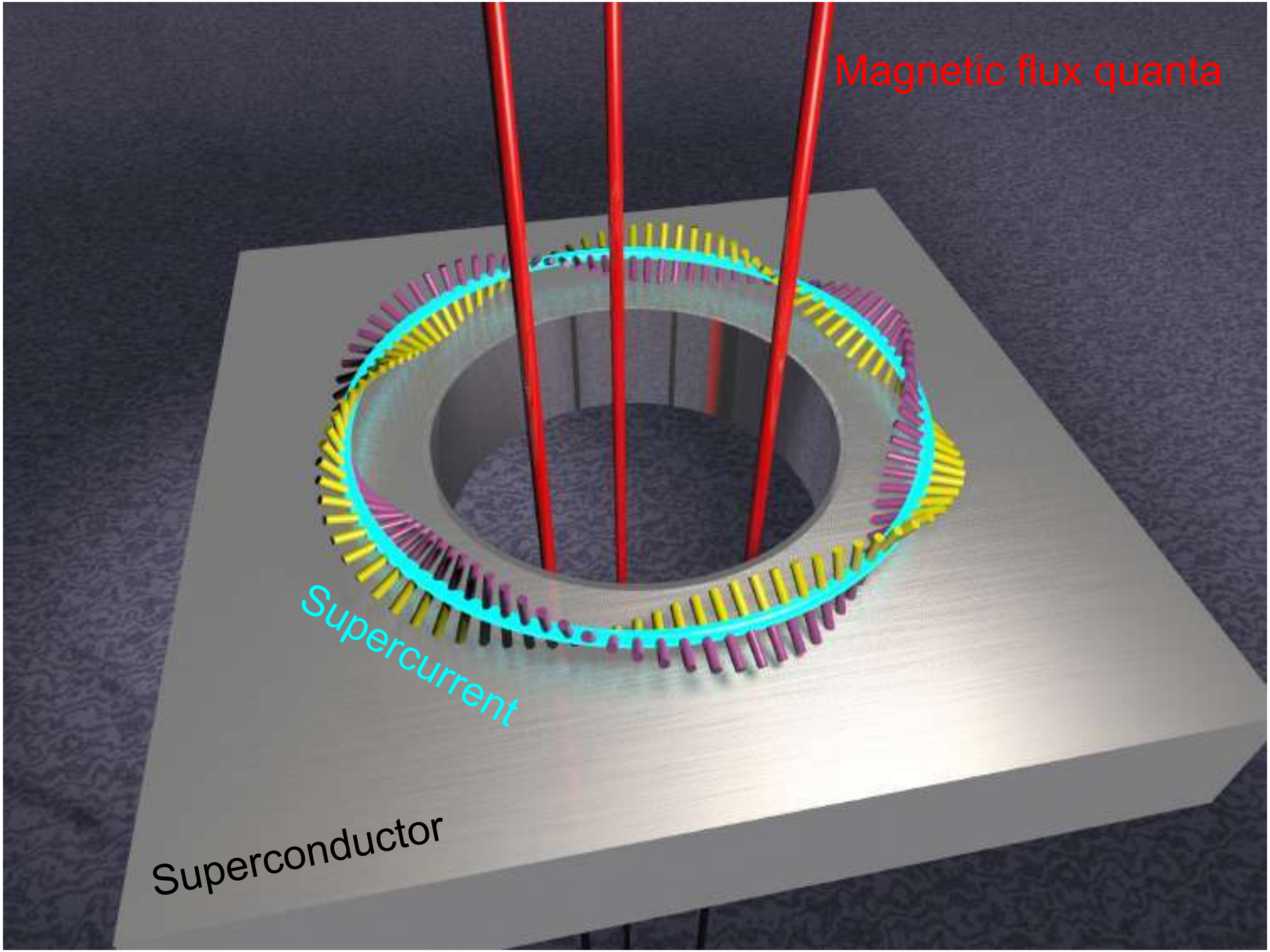
BAR MAGNET

"ARTIFICIAL" MAGNET
= Coil + battery

Superconductor with
persistent current!

Our house was a two storey house. I was in the kitchen cooking and suddenly the upstairs door was opened by Fritz. `Edith, Edith come, we have it. Come up, we have it.' And maybe the wind closed the door. I do not know what had happened upstairs. I left everything, ran up and, then, the door was opened in my face. On my forehead I had a bruise for a week. Fritz said `The equations are established. We have the solution. We can explain it.'

Edith London


Magnetic flux quanta
Supercurrent
Superconductor


SIEMENS
SIEMENS


Quantum coherent
wavefunction
SIEMENS

Fritz London (1936)

Rigidity

$$\mathbf{p} = m\mathbf{v} + q\mathbf{A} = 0$$

$$\Longrightarrow \mathbf{J} = nq\mathbf{v} = -\frac{nq^2\mathbf{A}}{m}$$

Maxwell equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \Longrightarrow \quad \nabla^2 \mathbf{B} = \frac{1}{\lambda^2}\mathbf{B}$$

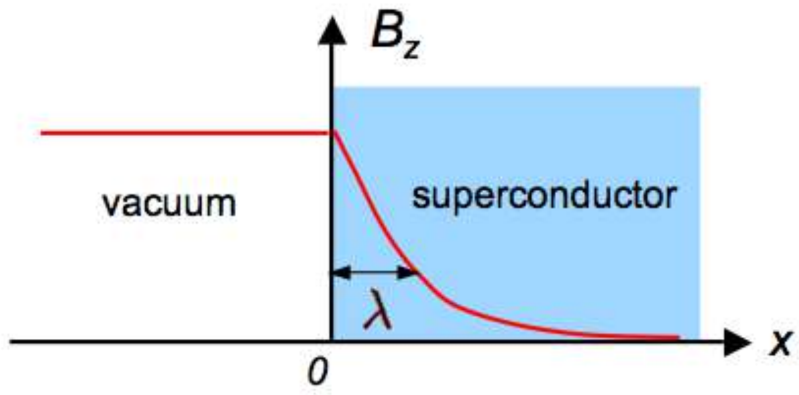
London penetration depth : $\lambda = \sqrt{m/\mu_0 n q^2}$

Fritz London (1936)

Magnetic field penetration in superconductor

in superconductor $$\frac{d^2 B_z}{dx^2} = \lambda^{-2} B_z$$

$$B_z(x) = B_z(0) e^{-x/\lambda}$$



Maxwell equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \implies \nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B}$$

London penetration depth : $\lambda = \sqrt{m/\mu_0 n q^2}$

$$\Psi(\underline{r}) = |\Psi(\underline{r})|\, e^{i\theta(\underline{r})}$$

ORDER PARAMETER

AMPLITUDE

PHASE

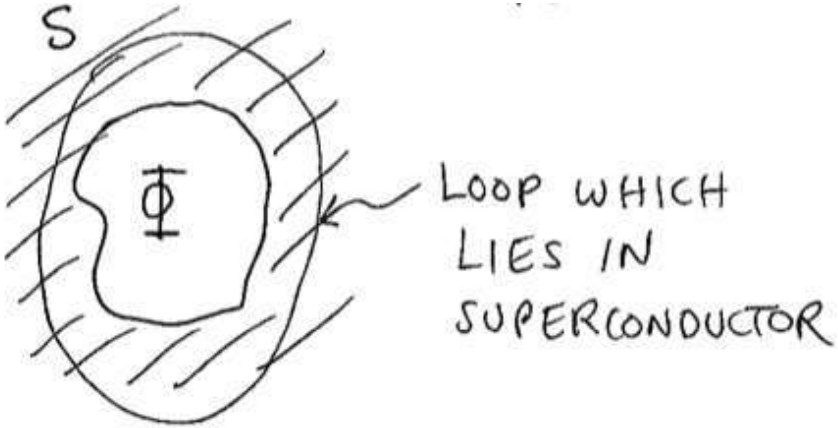
RIGID IN THE SUPERCONDUCTING STATE

$$m\underline{v} = \hbar \nabla \theta - q\underline{A}$$

GAUGE INVARIANT

$$\mathbf{j} = nq\mathbf{v} = \frac{nq}{m}(\hbar\nabla\theta - q\mathbf{A})$$

$$\oint \mathbf{j} \cdot \mathbf{ds} = 0$$



$$m\underline{v} = \hbar\nabla\theta - q\underline{A}$$

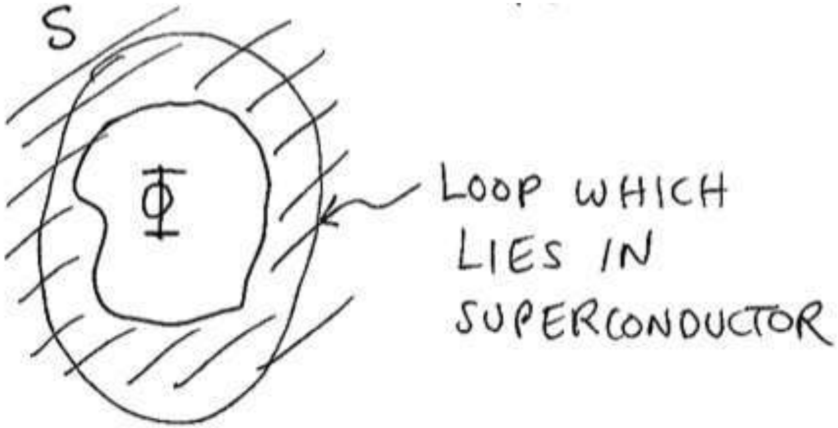
↑

GAUGE INVARIANT

$$\mathbf{j} = nq\mathbf{v} = \frac{nq}{m}(\hbar\nabla\theta - q\mathbf{A})$$

$$\oint \mathbf{j}\cdot\mathbf{ds} = 0 \quad \Longrightarrow \hbar \oint \nabla\theta \cdot d\mathbf{l} = q \oint \mathbf{A}\cdot d\mathbf{l}$$

$$\hbar\Delta\theta = q\int \mathbf{B}\cdot\mathbf{dS}$$



$$m\underline{v} = \hbar\nabla\theta - q\underline{A}$$

↑

GAUGE INVARIANT

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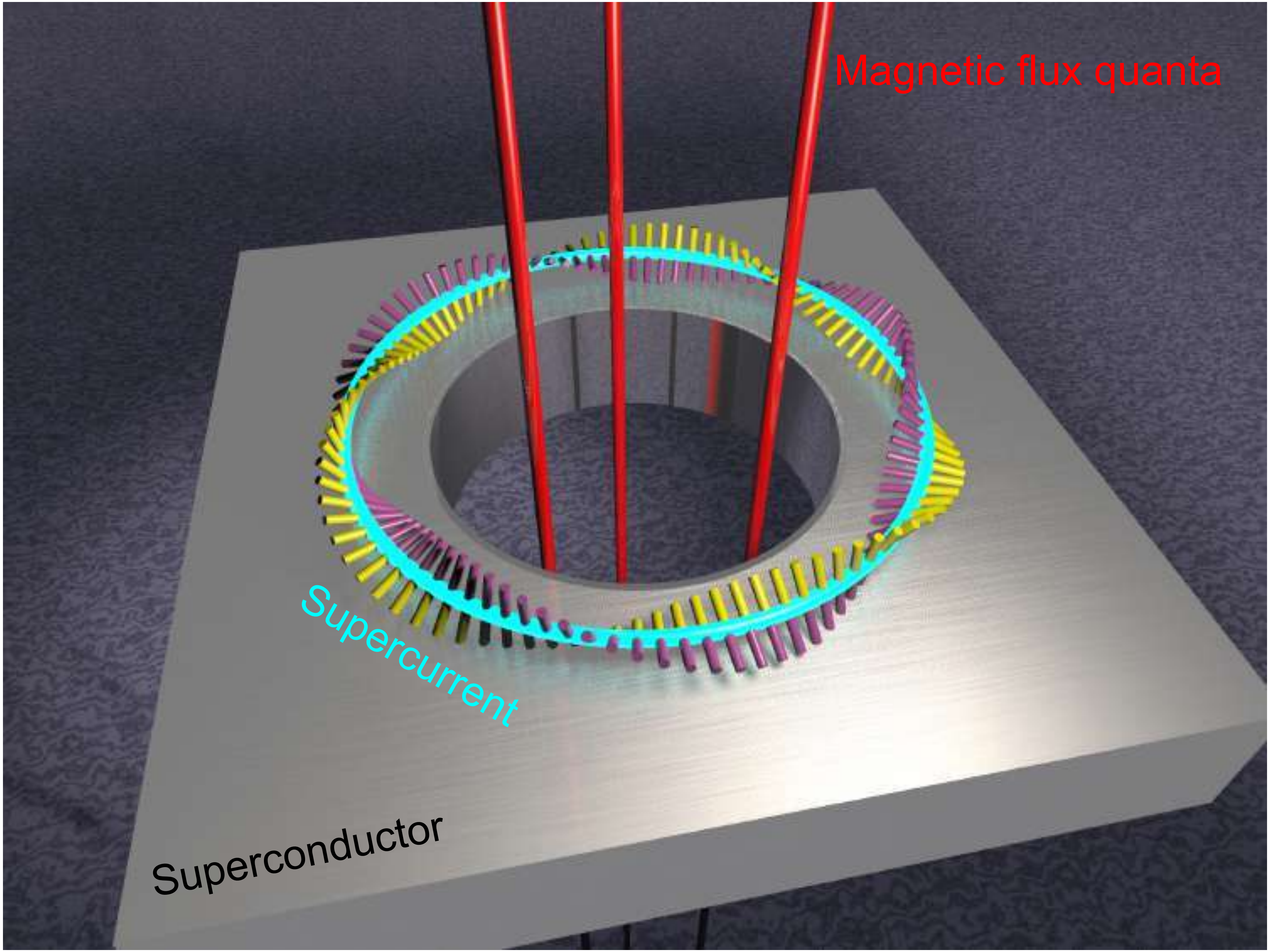
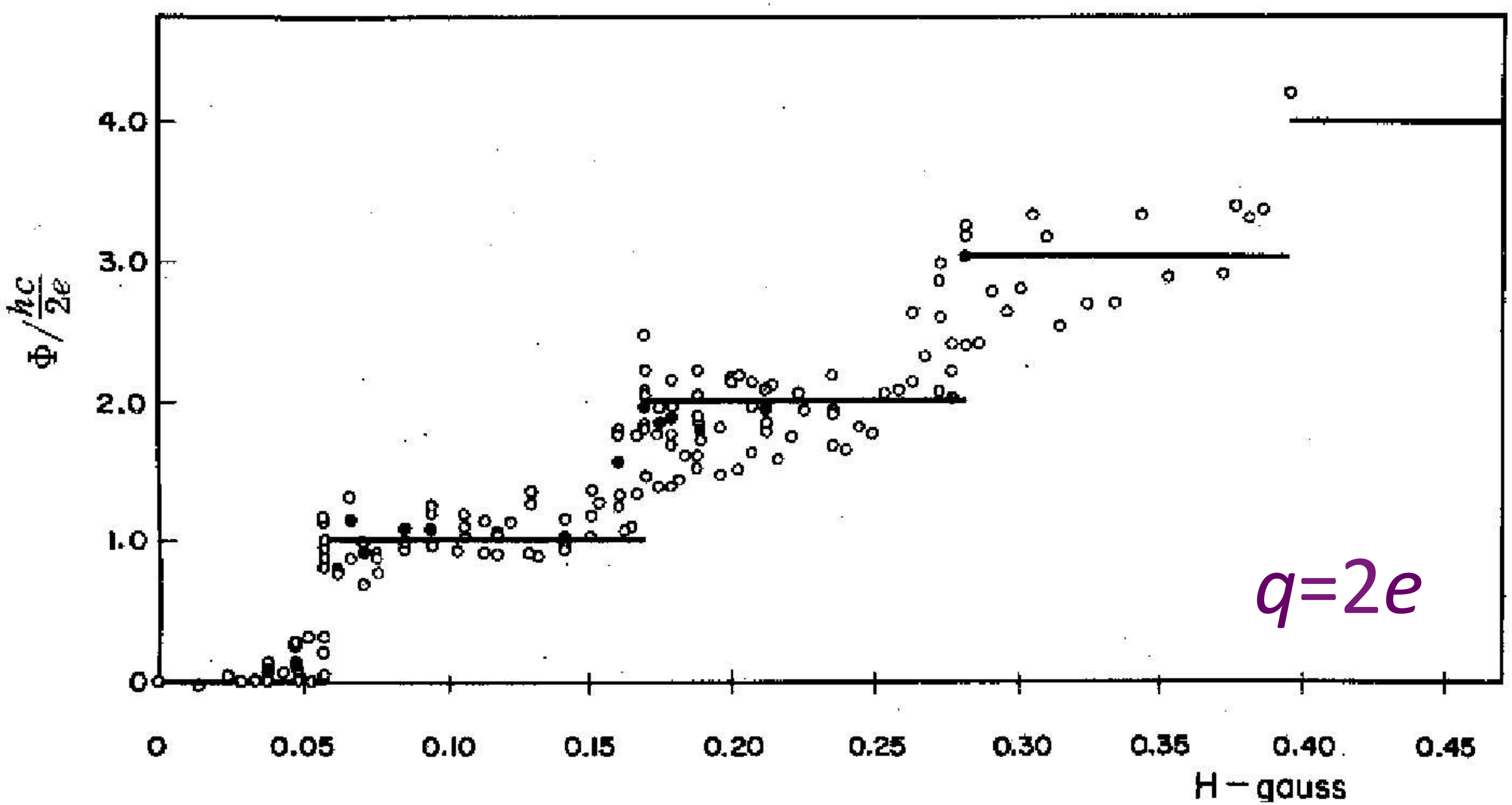
$$\hbar\Delta\theta = q \int \mathbf{B} \cdot \mathbf{dS}$$

$$\hbar(2\pi N) = q\Phi$$

Flux quantisation: $$\Phi = \frac{Nh}{q}$$

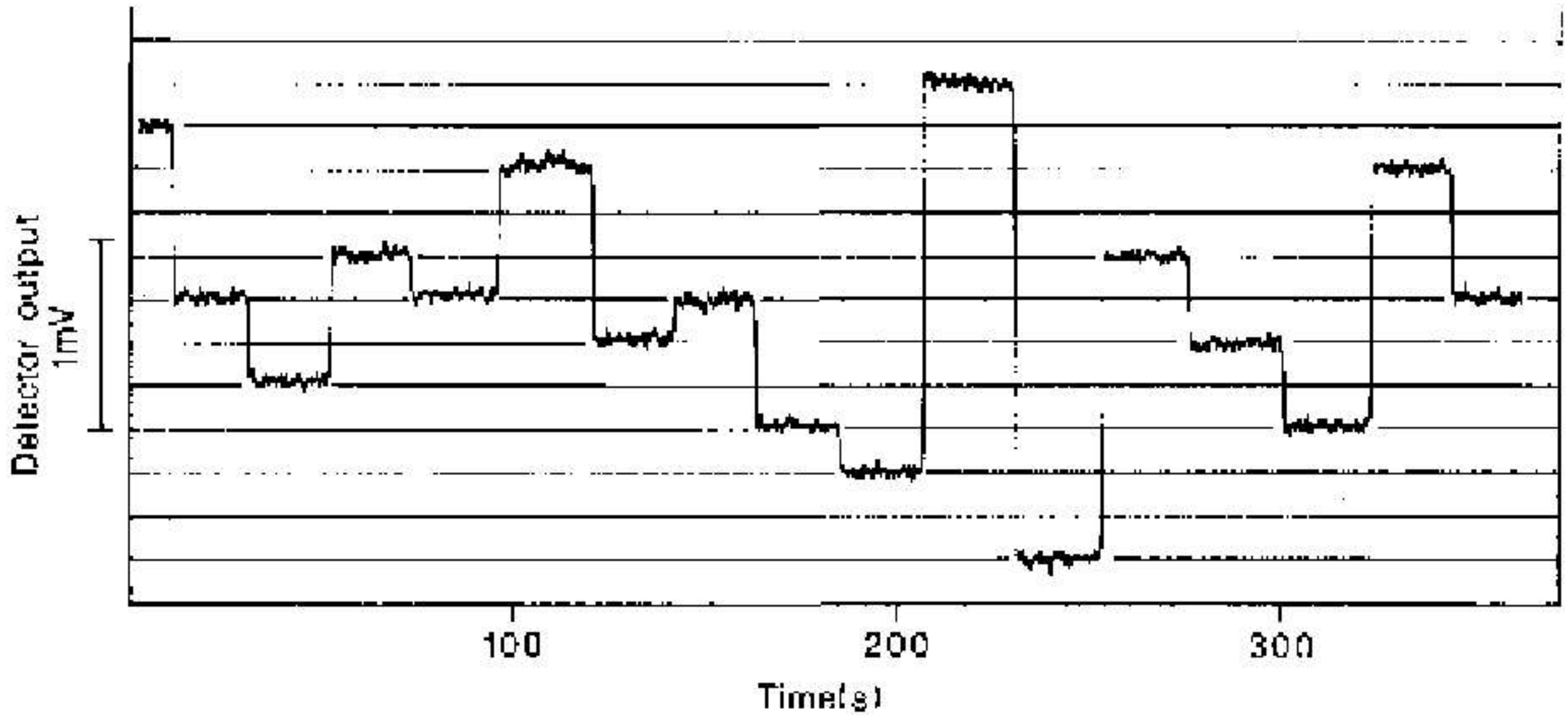

Magnetic flux quanta
Supercurrent
Superconductor

Measurements on a Sn cylinder: flux quantization!



1961: B. S. Deaver and W. M. Fairbank, Phys. Rev. Lett. 7, 43 (1961)

Measurements on a YBCO ring: flux quantization!



q=2*e*

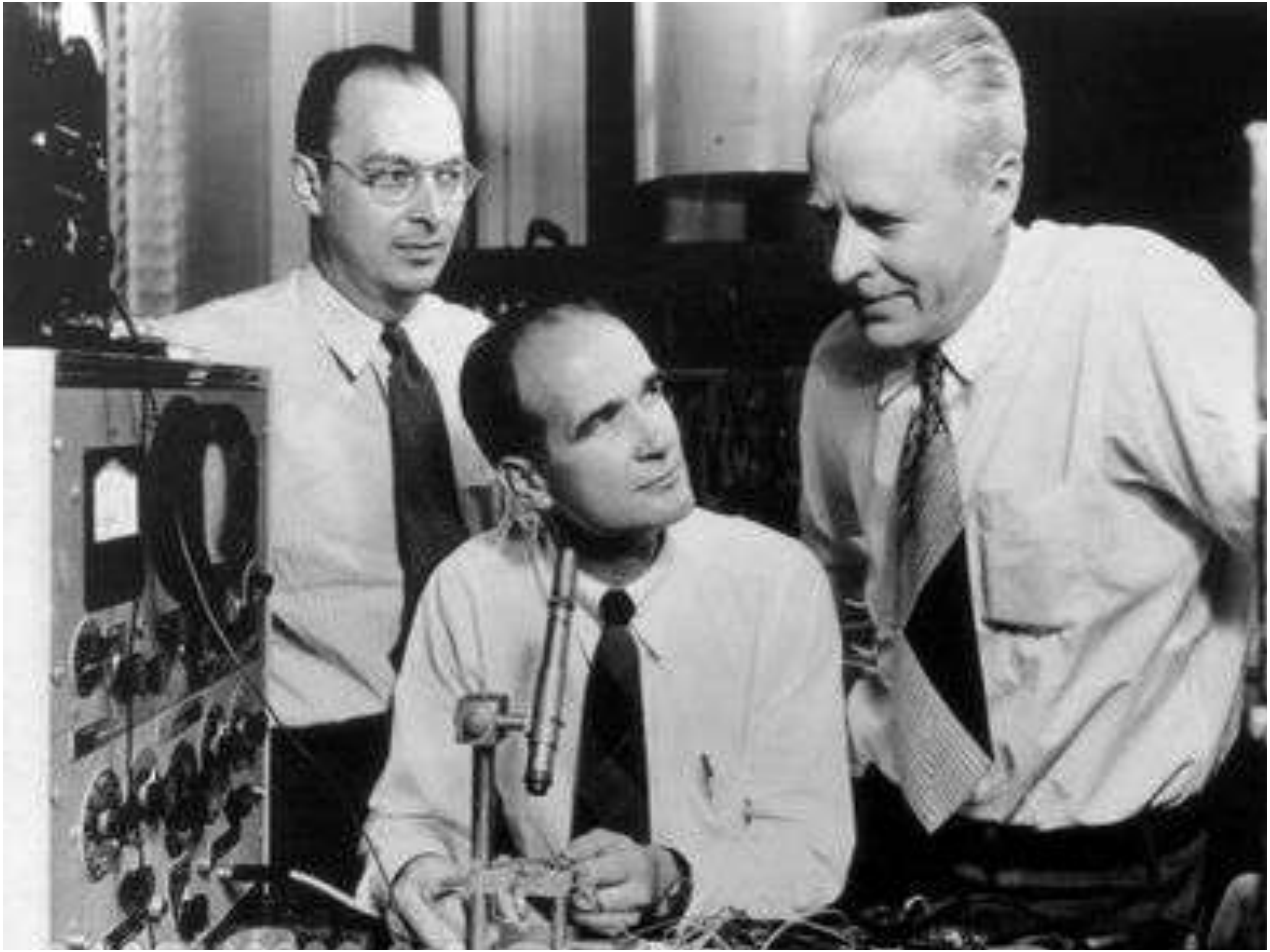
1987: C. E. Gough et al., Nature 326, 855 (1987). Flux jumps (0.97±0.04) h/(2e)

John Bardeen
(1908-1991)

Nobel prizes in physics

1956 The transistor (with Brattain and Shockley)

1972 BCS theory (with Cooper and Schrieffer)



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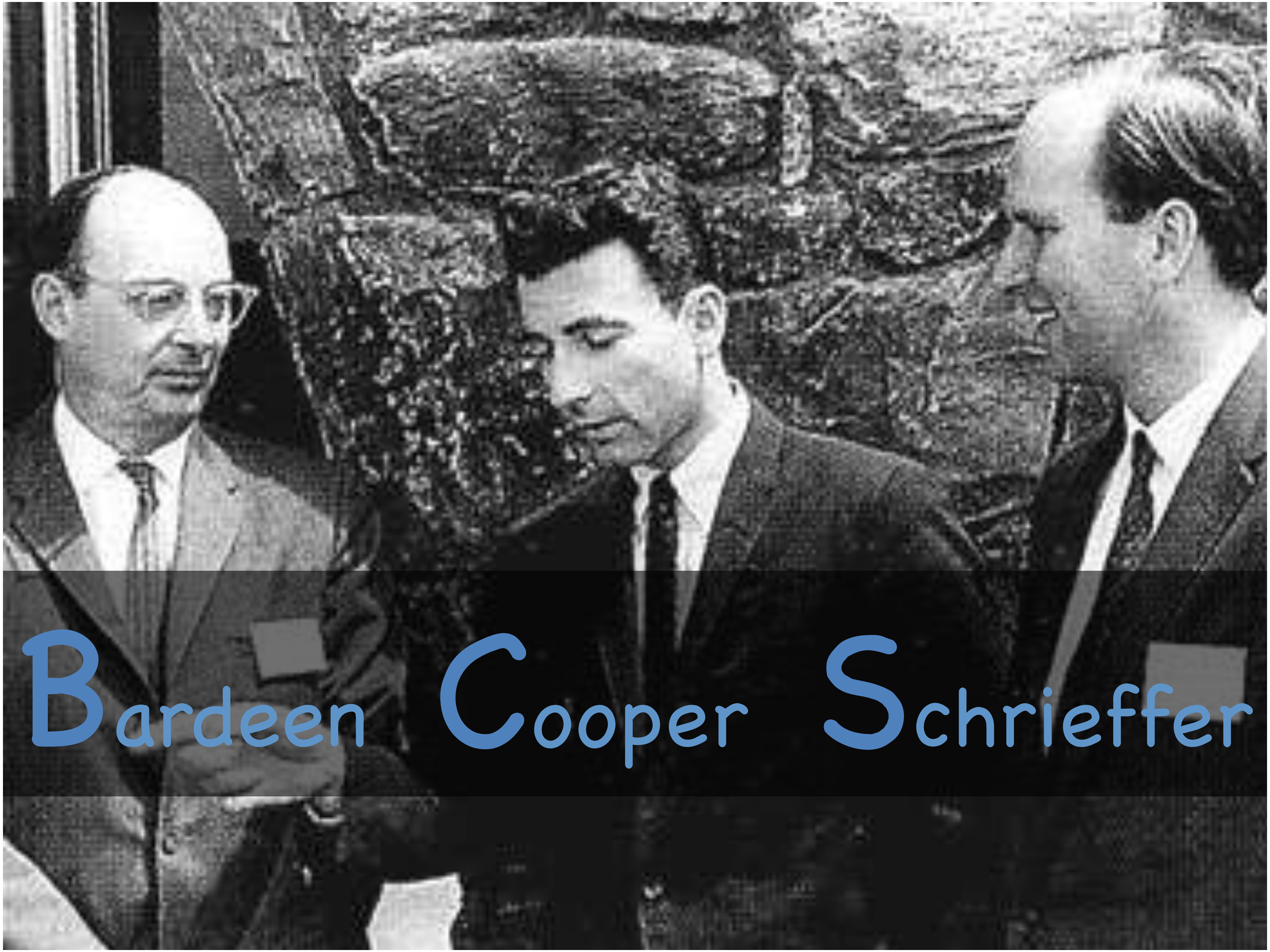
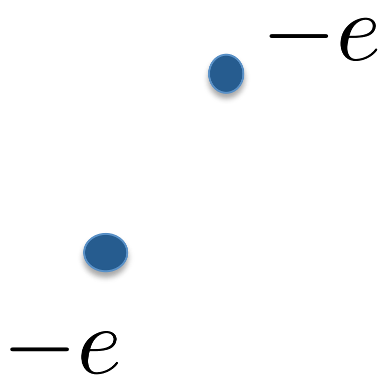

Bardeen Cooper Schrieffer

Superconductivity involves an unlikely pairing effect

Foreign & Commonwealth Office

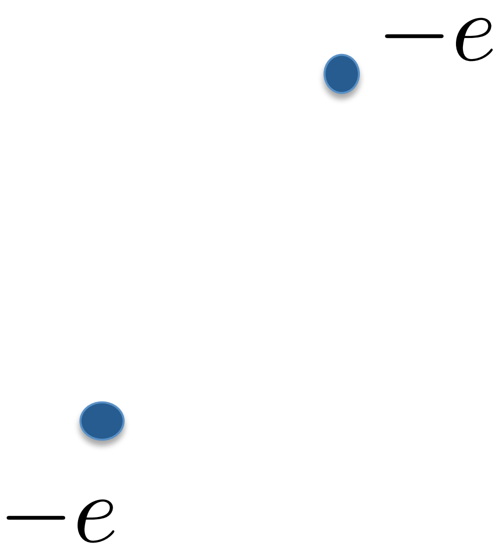
Home of Diplomacy

Superconductivity involves an unlikely pairing effect



$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

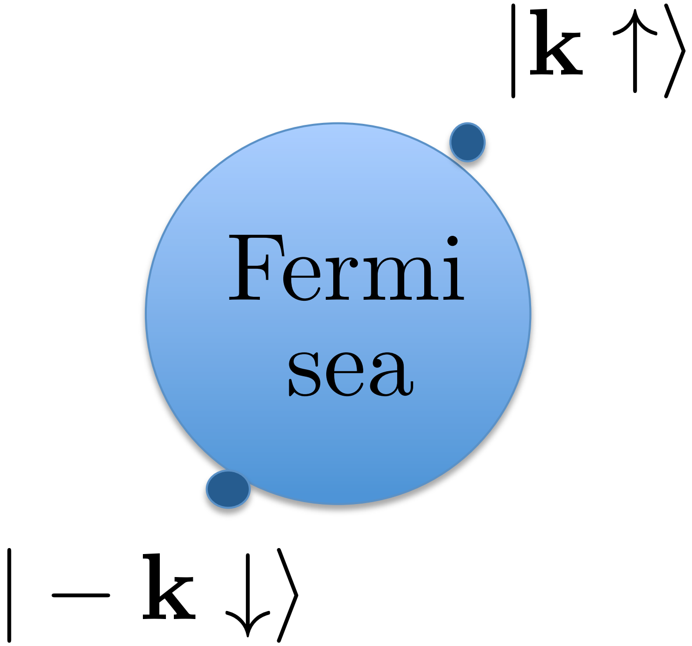
repulsive force



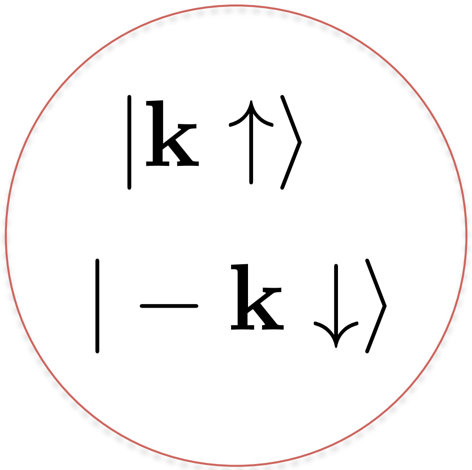
$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

repulsive force

Formation of Cooper pairs due to attractive interaction



Cooper pair:



$$P_{\text{tot}} = 0$$

$$L_{\text{tot}} = 0$$

$$S_{\text{tot}} = 0$$

Many body state: *coherent state* of Cooper pairs

$\hat{c}^{\dagger}_{\mathbf{k}\sigma}$ = creation operator, electron with momentum ***k***, spin σ

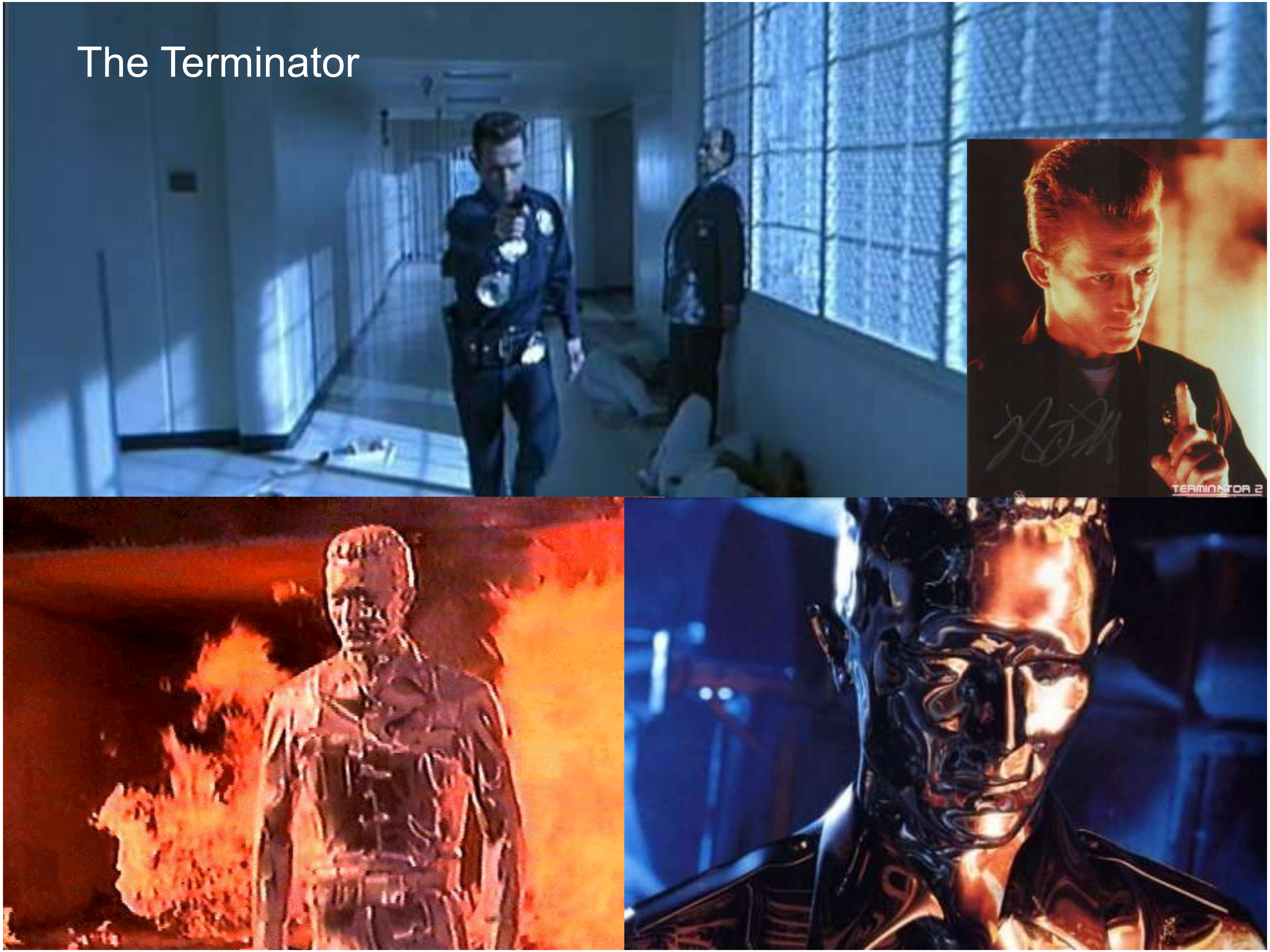
$\hat{c}_{\mathbf{k}\sigma}$ = annihilation operator, electron with momentum ***k***, spin σ

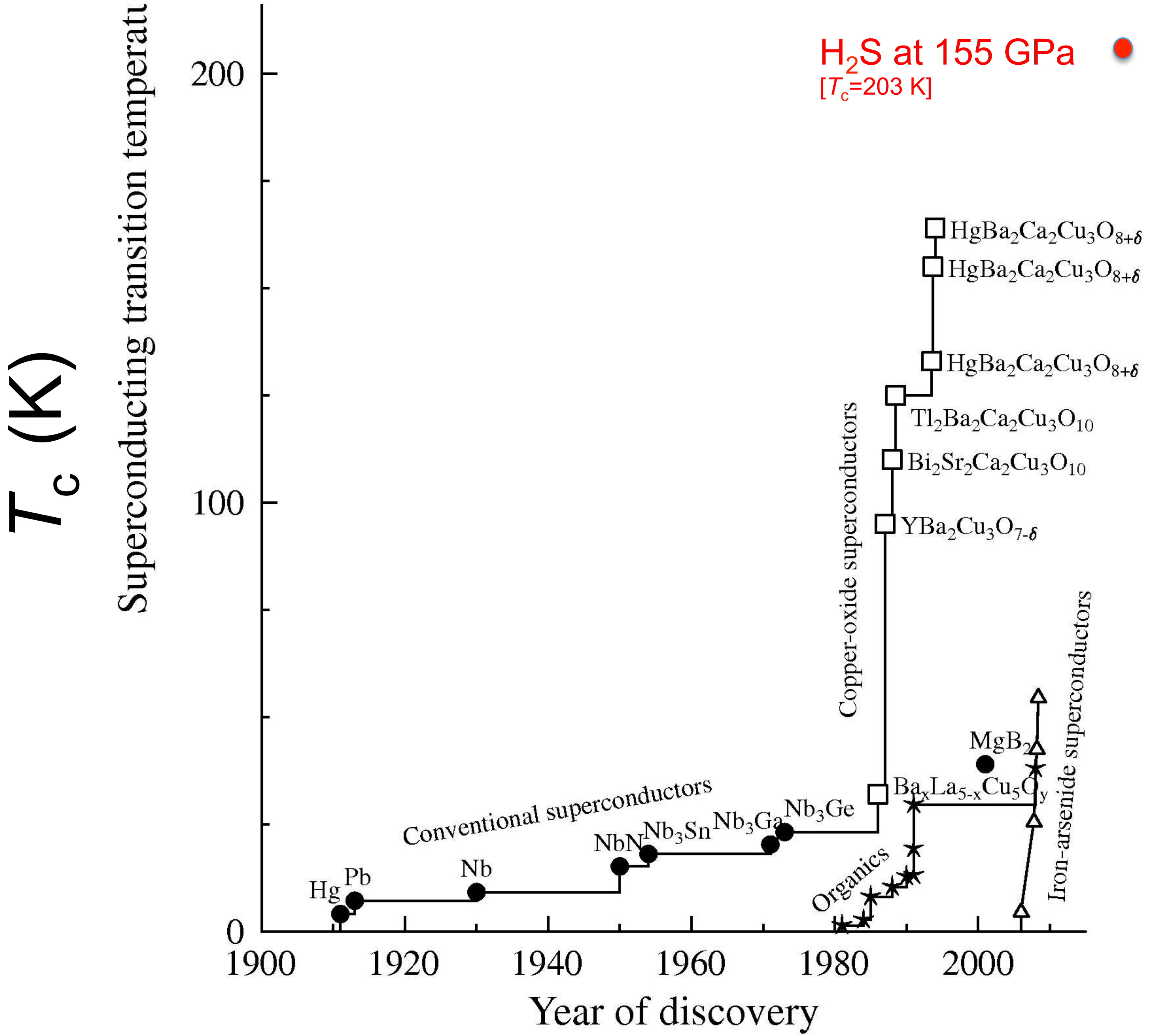
$\hat{P}^{\dagger}_{\mathbf{k}} = \hat{c}^{\dagger}_{\mathbf{k}\uparrow}\hat{c}^{\dagger}_{-\mathbf{k}\downarrow}$ = pair creation operator

$$|\text{Fermi sea}\rangle = \prod_{k<k_{\mathrm{F}}} \hat{P}^{\dagger}_{\mathbf{k}}|0\rangle$$

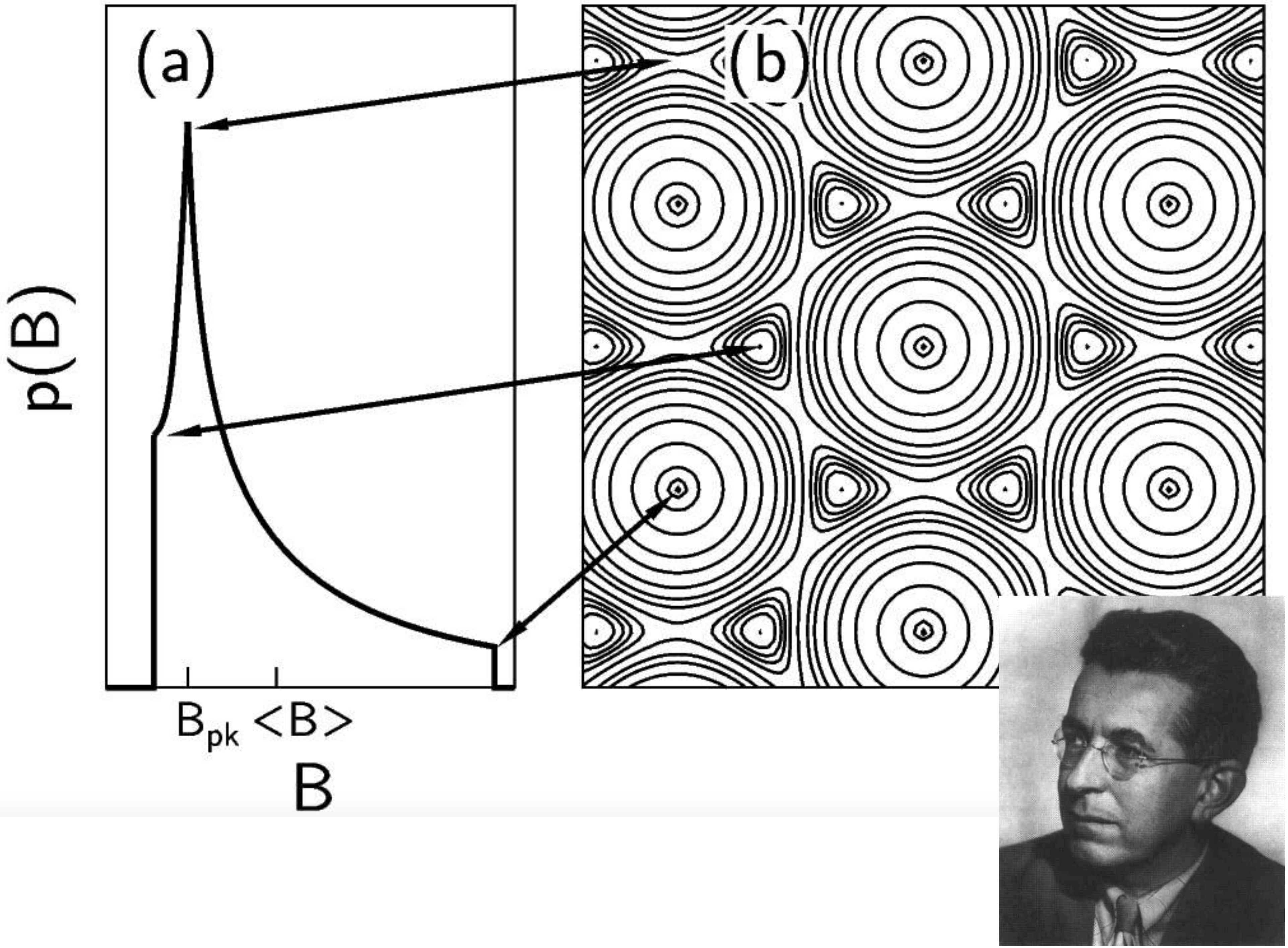
$$|\Psi_{\mathrm{BCS}}\rangle = \text{constant} \times \prod_{k} \exp(\alpha_k \hat{P}^{\dagger}_{\mathbf{k}})|0\rangle$$

The Terminator





H2S at 155 GPa
[Tc=203 K]
Tc (K)
Superconducting transition temperatu
Year of discovery
200
100
0
1900
1920
1940
1960
1980
2000
HgBa2Ca2Cu3O8+δ
HgBa2Ca2Cu3O8+δ
HgBa2Ca2Cu3O8+δ
Tl2Ba2Ca2Cu3O10
Bi2Sr2Ca2Cu3O10
YBa2Cu3O7-δ
Copper-oxide superconductors
MgB2
BaxLa5-xCu5Oy
Iron-arsenide superconductors
Conventional superconductors
Hg
Pb
Nb
NbN
Nb3Sn
Nb3Ga
Nb3Ge
Organics


(a)
(b)
p(B)
B$_{pk}$ <B>
B